General Physics III
PHY 103

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Surface Tension

It has been observed that a fisherman's boat is often surrounded by fallen leaves that are lying on the water. The boat floats, because it is partially immersed in the water and the resulting buoyant force balances its weight. The leaves, however, float for a different reason. They are not immersed in the water, so the weight of a leaf is not balanced by a buoyant force. Instead, the force balancing a leaf's weight arises because of the surface tension of the water. Surface tension is a property that allows the surface of a liquid to behave somewhat as a trampoline does. When a person stands on a trampoline, the trampoline stretches downward a bit and, in so doing, exerts an upward elastic force on the person. This upward force balances the person's weight. The surface of the water behaves in a similar way. In Figure 1, for instance, you can see the indentations in the water surface made by the feet of an insect known as a water strider, because it can stride or walk on the surface just as a person can walk on a trampoline.

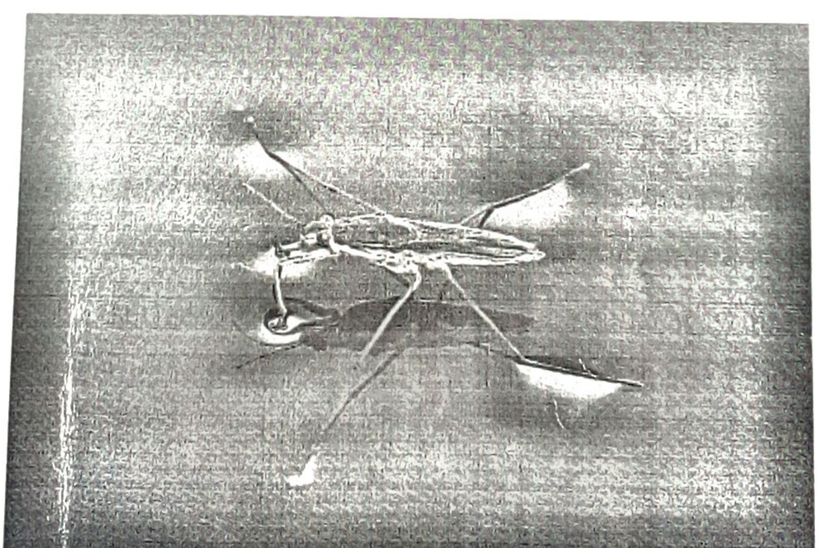
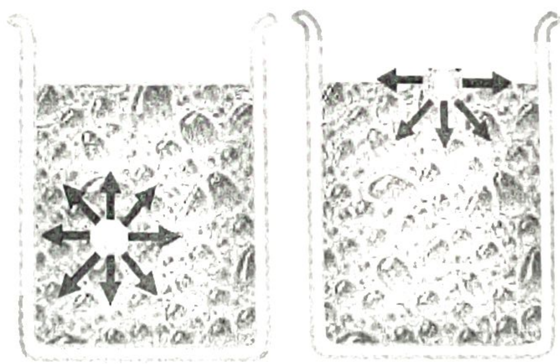


Figure 1 The surface tension of the water allows the insect to walk on the water without sinking.

Figure 2 illustrates the molecular basis for surface tension by considering the attractive forces that molecules in a liquid exert on one another. Part *a* shows a molecule within the bulk liquid, so that it is surrounded on all sides by other molecules. The surrounding molecules attract the central molecule equally in all directions, leading to a zero net force. In contrast, part *b* shows a molecule in the surface. Since there are no molecules of the liquid above the surface, this molecule experiences a net attractive force pointing toward the liquid interior. This net attractive force causes the liquid surface to contract toward the interior until repulsive collisional forces from the other molecules halt the contraction at the point when the surface area is a minimum. If the liquid is not acted upon by external forces, a liquid sample forms a sphere, which has the minimum surface area for a given volume. Nearly spherical drops of water are a familiar sight, for example, when the external forces are negligible.



(a) (b)

Figure 2 (a) A molecule within the bulk liquid. (b) A molecule in the surface

Definition of Surface Tension

The surface tension γ is the magnitude F of the force exerted parallel to the surface of a liquid divided by the length L of the line over which the force acts:

$$\gamma = F/L \ldots\ldots (2)$$

The SI Unit of Surface Tension is Newton/meter i.e. N /m

Surface tension has the dimension of force per unit length, or of energy per unit area. The two are equivalent but when referring to energy per unit of area, people use the term surface energy which is a more general term in the sense that it applies also to solids and not just liquids.

Surface Tension Examples

Walking on water Small insects such as the **water strider** can walk on water because their weight is not enough to penetrate the surface.	**Floating a needle** If carefully placed on the surface, a small needle can be made to float on the surface of water even though it is several times as dense as water. If the surface is agitated to break up the surface tension, then needle will quickly sink.
Don't touch the tent! Common tent materials are somewhat rainproof, in that the surface tension of water will bridge the pores in the finely woven material. But if you touch the tent material with your finger, you break the surface tension and the rain will drip through.	**Soaps and detergents** Help the cleaning of clothes by lowering the surface tension of the water so that it more readily soaks into pores and soiled areas.
Clinical test for jaundice Normal urine has a surface tension of about 66 dynes/cm but if bile is present (a test for jaundice), it drops to about 55. In the Hay test, powdered sulfur is sprinkled on the urine surface. It will float on normal urine, but sink if the S.T. is	**Washing with cold water** The major reason for using hot water for washing is that its surface tension is lower and it is a better wetting agent. But if the detergent lowers the surface tension, the heating may be unnecessary.

lowered by the bile.	
Surface tension disinfectants Disinfectants are usually solutions of low surface tension. This allow them to spread out on the cell walls of bacteria and disrupt them. One such disinfectant, S.T.37, has a name which points to its low surface tension compared to the 72 dynes/cm for water.	

Note:
(1) It depends only on the nature of liquid and is independent of the area of surface or length of line considered.
(2) It is a scalar as it has a unique direction which is not to be specified.
(3) Dimension: $[MT^{-2}]$
(4) Units: N/m (S.I.) and $Dyne/cm$ [C.G.S.]
(5) It is a molecular phenomenon and its root cause is the electromagnetic forces.

Factors Affecting Surface Tension.

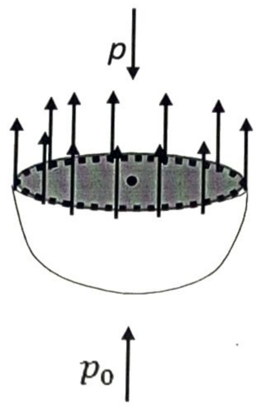
Temperature: The surface tension of liquid decreases with rise in temperature. The surface tension of liquid is zero at its boiling point and it vanishes at critical temperature.

Examples: (i) Hot soup tastes better than the cold soup.
(ii) Machinery parts get jammed in winter.

Impurities:

(i) Surface tension decreases when partially soluble impurities such as soap, detergent, Dettol, phenol etc are added in water.
(ii) Surface tension increases when highly soluble impurities such as salt is added in water.
(iii) When dust particles or oil spreads over the surface of water, its surface tension decreases.

Pressure Difference between bubble and drop



The surface tension causes a pressure difference between the inside and outside of a soap bubble or a liquid drop. A soap bubble consists of two spherical surface films with a thin layer of liquid between them.

Let p = pressure exerted by the upper half, and p_o= external pressure.

Therefore, force exerted due to surface tension = $2(2\pi r\gamma)$ (the "2" is for two surfaces). In equilibrium the forces must be equal:

Therefore $F_p = F_\gamma + Fp_0$

Hence, $\pi r^2 p = 2(2\pi r\gamma) + \pi r^2 po.$

So excess pressure is, P – Po

i.e.$\pi r^2 p = 2(2\pi r\gamma) + \pi r^2 p_o$

$(P - P_o)\pi r^2 = 4\pi r\gamma$

Divide through by πr

$P - P_o = \frac{4\gamma}{r}$........

For a liquid drop, the difference is that there is only one surface and so, excess pressure =

i.e.$\pi r^2 p = (2\pi r\gamma) + \pi r^2 p_o$

$(P - P_o)\pi r^2 = 2\pi r\gamma$

Divide through by πr

$$P - P_o = \frac{2\gamma}{r} \ldots\ldots\ldots$$

This equation determines the excess pressure to maintain the bubble, where r is the radius of curvature

For any curved liquid surface or meniscus whose r is the radius of curvature and γ is the surface tension, and the angle of contact is zero, then the excess pressure is given by

$$P - P_o = \frac{2\gamma}{r} \ldots\ldots$$

If the contact angle Θ, then the excess pressure is given by

$$P - P_o = \frac{2\gamma \cos\theta}{r} \ldots\ldots$$

Example 1

A student, using a circular loop of wire and a pan of soapy water, produces a soap bubble whose radius is 1.0 mm. The surface tension of the soapy water is $\gamma = 2.5 \times 10^{-2}$N/m. Determine the pressure difference between the inside and outside of the bubble. (b)The same soapy water is used to produce a spherical droplet whose radius is one-half that of the bubble, or 0.50 mm. Find the pressure difference between the inside and outside of the droplet.

Solution

(a) The pressure difference, $P - P_o$, between the inside and outside of the soap bubble is given by

$$P - P_o = \frac{4\gamma}{r}$$

$$\frac{4(2.5 \times 10^{-2} N/m)}{1.0 \times 10^{-3} m} = 1.0 \times 10^2\ N/m^2$$

(b) The pressure difference $P - P_o$, between the inside and outside of the drop is given by

$$P - P_o = \frac{2\gamma}{r}$$

$$\frac{2(2.5 \times 10^{-2} N/m}{0.50 \times 10^{-3} m} = 1.0 \times 10^2\ N/m^2$$

Cohesion and Adhesion

Molecules in liquid state experience strong intermolecular attractive forces. When those forces are between like molecules, they are referred to as cohesive forces. This force is lesser in liquids and least in gases. For example, the molecules of a water droplet are held together by cohesive forces, and the especially strong cohesive forces at the surface constitute *surface tension.*

When the attractive forces are between unlike molecules, they are said to be adhesive forces. The adhesive forces between water molecules and the walls of a glass tube are stronger than the cohesive forces which lead to an upward turning meniscus at the walls of the vessel and contribute to *capillary action.*

Capillarity

It is a verifiable fact that, if a tube of very narrow bore (called capillary) is dipped in a liquid, it is found that the liquid in the capillary either ascends or descends relative to the surrounding liquid. This phenomenon is called capillarity. It is a consequence of surface tension.

The root cause of capillarity is the difference in pressures on two sides of (concave and convex) curved surface of liquid.

Examples of capillarity:

(i) Ink rises in the fine pores of blotting paper leaving the paper dry.

(ii)A towel soaks water.

(iii) Oil rises in the long narrow spaces between the threads of a wick.

(iv) Wood swells in rainy season due to rise of moisture from air in the pores.

(v) Ploughing of fields is essential for preserving moisture in the soil.

(vi) Sand is drier soil than clay. This is because holes between the sand particles are not so fine as compared to that of clay, to draw up water by capillary action.

Angle of Contact:

Angle of contact between a liquid and a solid is defined as the angle enclosed between the tangents to the liquid surface and the solid surface inside the liquid, both the tangents being drawn at the point of contact of the liquid with the solid.

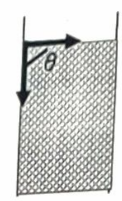
$\theta < 90°$

$F_a > \frac{F_c}{\sqrt{2}}$

Concave meniscus.
Liquid wets the solid surface

$\theta = 90°$

$F_a = \frac{F_c}{\sqrt{2}}$



plane meniscus.
Liquid does not wet the solid surface

$\theta > 90°$

$F_a < \frac{F_c}{\sqrt{2}}$

convex meniscus.
Liquid does not wet the solid surface

Note:

(i) Its value lies between 0° and 180°, $\theta = 0^0$ for pure water and glass, $\theta = 8^0$ for tap water and glass, $\theta = 90^0$ for water and silver $\theta = 138^0$ for mercury and glass, $\theta = 160^0$ for water and chromium.

(ii) It is particular for a given pair of liquid and solid. Thus the angle of contact changes with the pair of solid and liquid.

(iii) It does not depend upon the inclination of the solid in the liquid.

(iv) On increasing the temperature, angle of contact decreases.

(v) Soluble impurities increase the angle of contact.

Ascent Relation

When one end of capillary tube of radius r is immersed into a liquid of density ρ which wets the sides of the capillary tube (water and capillary tube of glass), the shape of the liquid meniscus in the tube becomes concave upwards.

Let us ask this question, why should water rise up inside the tube? It is an effect of the surface tension at the top of the water column, particularly where it meets the glass wall.

Each water surface molecule exerts forces on those near it. Since there is equilibrium the lastwater molecule must also have a force exerted on it by the glass molecule near it. Therefore, all around the top of the water, the glass is exerting a force on the water. Because, water wets glass, this force is a **vertical** force. So that is why the water rises in the tube: because the glass is pulling it up. The length of the line of contact between the water and the glass is 2π times the radius of tube, so the magnitude of the upward force is

$$= \gamma \text{ x } (2\pi \text{radius of tube})$$
$$= 2\pi r\gamma$$

Why does the water not keep rising indefinitely?

The answer is that the higher the column the more the weight of the water in the column pulls it back. Thus there is a downward force equal to $\rho(\pi r^2 h)\, g$

The two forces are in equilibrium so

$$2\pi r\gamma = \rho\pi r^2 hg$$

and, therefore, for this situation, where the water wets the glass completely, the final

Pressure due to liquid column = pressure difference due to surface tension

height of thewater column can be written as

$$h = \frac{2\gamma}{\rho g r}$$

Or

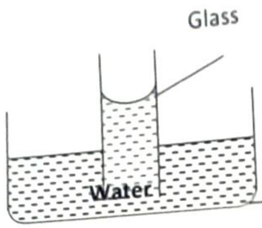
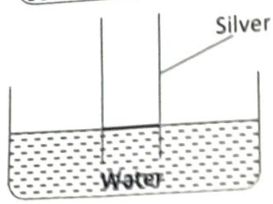
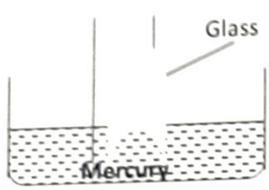
$$h = \frac{2\gamma \text{Cos}\theta}{\rho g r}$$

where Θ is the angle of contact.

Note also,

(i) The capillary rise depends on both the nature of liquid and solid *i.e.* on γ, ρ, θ and R.

(ii) Capillary action for various liquid-solid pair.

	Meniscus	Angle of contact	Level
	Concave	$\theta < 90°$	Rises
	Plane	$\theta = 90°$	No rise or fall
	Convex	$\theta > 90°$	Fall

(iii) For a given liquid and solid at a given place

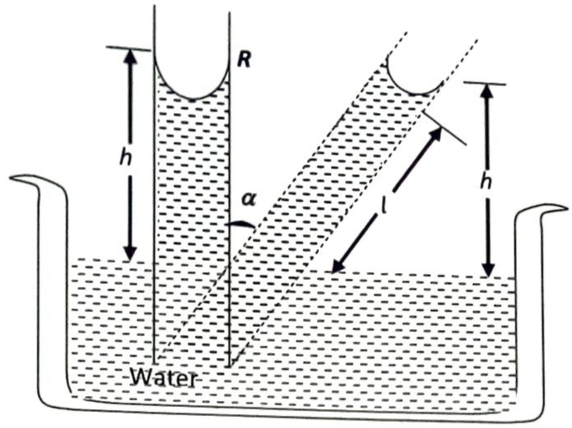
$h \propto \frac{1}{r}$ [$As\ \gamma, \theta, \rho$ *and g are constant*]

i.e. the lesser the radius of capillary the greater will be the rise and vice-versa.

(iv) If the weight of the liquid contained in the meniscus is taken into consideration then more accurate ascent formula is given by

$$h = \frac{2\gamma \cos\theta}{r\rho g} - \frac{r}{3}$$

(v) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical, then the vertical height of the liquid column remains same whereas the length of liquid column (l) in the capillary tube increases.



Work Done in Blowing a Liquid Drop or Soap Bubble.

(i) If the initial radius of liquid drop is r_1 and final radius of liquid drop is r_2 then
$W = \gamma \times$ Increment in surface area

For a spherical bubble, the volume and surface area are given simply by

$$V = \frac{4}{3}\pi R^3 \rightarrow dV \approx 4\pi R^2 dR,$$

and

$$A = 4\pi R^2 \rightarrow dA \approx 8\pi R dR.$$

$W = \gamma \times 4\pi[r_2{}^2 - r_1{}^2]$ [Drop has only one free surface]

(ii) In case of soap bubble

$W = \gamma \times 8\pi[r_2{}^2 - r_1{}^2]$ [Bubble has two free surfaces]

Splitting of Bigger Drop.

When a drop of radius R splits into n smaller drops, (each of radius r) then surface area of liquid increases. Hence the work is to be done against surface tension.

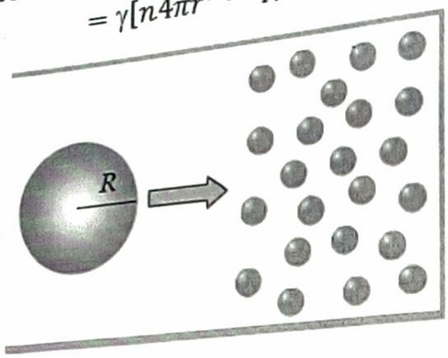
Since the volume of liquid remains constant, therefore $\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$

$\therefore\ R^3 = nr^3$

Work done $= \gamma \times \Delta A$

$= \gamma$[Total final surface area of n drops – surface area of big drop]

$= \gamma[n4\pi r^2 - 4\pi R^2]$

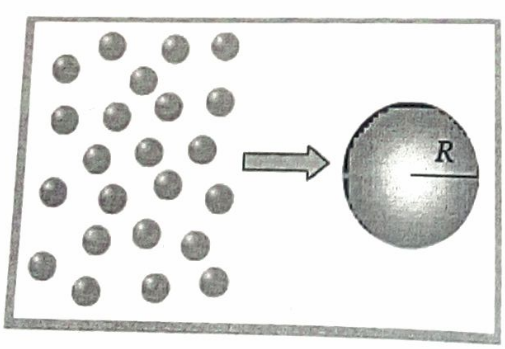


Formation of Bigger Drop

If n small drops of radius r coalesce to form a big drop of radius R then surface area of the liquid decreases.

Amount of surface energy released = Initial surface energy – final surface energy

$$E = n4\pi r^2 T - 4\pi R^2 T$$



NOTE:

(1) Formation of double bubble: If r_1 and r_2 are the radii of smaller and larger bubble and P_0 is the atmospheric pressure, then the pressure inside them will be

$$P_1 = P_0 + \frac{4T}{r_1} \text{ and } P_2 = P_0 + \frac{4T}{r_2}$$

Now as $r_1 < r_2 \therefore P_1 > P_2$

So for interface $\Delta P = P_1 - P_2 = 4T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ i

As excess pressure acts from concave to convex side, the interface willbe concave towards the smaller bubble and convex towards larger bubbleand if r is the radius of interface.

$$\Delta P = \frac{4T}{r}$$ ii

From (i) and (ii) $\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$

Therefore,

Radius of the interface $r = \frac{r_1 r_2}{r_2 - r_1}$

Questions

1. Water rises to a height of *10cm* in a capillary tube and mercury falls to a depth of 3.5*cm* in the same capillary tube. If the density of mercury is 13.6 *gm/cc* and its angle of contact is 135^0 and density of water is 1 *gm/cc* and its angle of contact is 0^0, what is the ratio of surface tensions of the two liquids? (Take cos135 = 0.7)

Solution:

$$h = \frac{2T\cos\theta}{rdg} \qquad \therefore \frac{h_W}{h_{Hg}} = \frac{T_W}{T_{Hg}} \frac{\cos\theta_W}{\cos\theta_{Hg}} \frac{d_{Hg}}{d_W}$$ [as r and g are constants]

$$\Rightarrow \frac{10}{3.5} = \frac{T_W}{T_{Hg}} \cdot \frac{\cos 0^o}{\cos 135} \frac{13.6}{1} \Rightarrow \frac{T_W}{T_{Hg}} = \frac{10 \times 0.7}{3.5 \times 13.6} = \frac{20}{136} = \frac{5}{34}$$

Therefore, the ratio of the liquids =5:34

2. Water rises in a vertical capillary tube up to a height of 2.0 *cm*. If the tube is inclined at an angle of 60^0 with the vertical, then up to what length will the water rise in the tube?

Solution:

The height upto which water will rise

$$l = \frac{h}{\cos\alpha} = \frac{2cm}{\cos 60}$$

=2cm/0.5 = 4cm.

3. If the surface tension of water is 0.06 *N/m*, then the capillary rise in a tube of diameter 1*mm* is what?
(Take $\theta = 0^0$, $d = 10^3 kg/m^3$, $g = 9.8\ m/s^2$)
Solution:

$$h = \frac{2T\cos\theta}{rdg}, \quad [\theta = 0,\ r = \frac{1}{2}mm = 0.5\times10^{-3}m$$

$$h = \frac{2\times0.06\times\cos\theta}{0.5\times10^{-3}\times10^{3}\times9.8} = 0.0244m = 2.44cm$$